cross-sectional idealization (no overlap) used. BI-STEM tubes inevitably have some overlap in reality (Fig. 1) and are interlocked in some fashion, both circumstances providing added strength. Furthermore, there is experimental evidence[6] which also shows that Eq. (9) is conservative. The fact that the actual cross section is typically greater than the one used in the present theory can be taken into account by introducing the section modulus Z as basis, rather than the radius r. The section modulus for the cross section used to derive Eq. (9) is

$$Z = \pi r^2 t \tag{13}$$

such that Eq. (9), with $\nu = 0.3$, can be expressed as

$$M_{cr} = 0.38EZ\frac{t}{d} \tag{14}$$

where $d = 2r$ is the tube diameter. In this form, with the section modulus Z of the actual cross section, the equation is suitable for critical bending moment prediction for double-slit tubing with overlapping and interlocking.

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A Simple Estimation Procedure of Roll-Rate Derivatives for Finned Vehicles

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Introduction

SOME of the important aspects that warrant consideration in the prediction of the roll-rate derivatives (C_{lp} and $C_{l\delta}$) are 1) normal-load distribution over the fin planform, 2) choice of the proper fin-body interference factor, and 3) the cruciform configuration effects. Barrowmann et al.[1] have brought out a simple and logical approach for the estimation of C_{lp} and $C_{l\delta}$. However, their approach considers a uniform load, $CN_{\alpha f}$, over the fin in the subsonic regime which may lead to overprediction of the roll-rate derivatives. Even in the supersonic regime the use[1] of Busemann's higher-order theory for small incidence may lead to slight overprediction as studied by Oberkampf.[2] Further, many investigators[1-4] do not appear to have considered the cruciform effects, as brought out by Adams and Dugan.[5]

The present Note briefly outlines a more generalized formulation for computing $C_{l\delta}$ and C_{lp}, discusses the adaption of proper fin-body and fin-fin interference effects, and presents a comparison of the present calculations with experimental data which are found to be in good agreement. However, the present approach overpredicts C_{lp} at subsonic speeds in certain cases.

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Analysis

Roll Moment Calculation

The roll moment coefficient for planar fin (suffix-) can be derived as

$$C_{l-} = \frac{2}{SD}\int\int_{A'} \Delta C_p \alpha(\xi)\xi \mathrm{d}x\mathrm{d}\xi \tag{1}$$

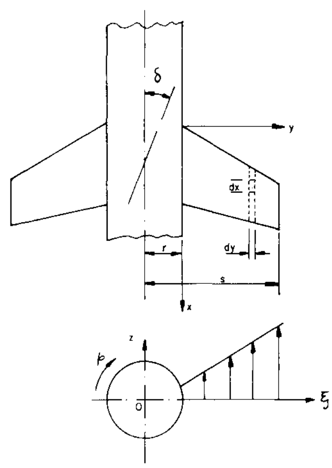

Fig. 1a Roll parameters.

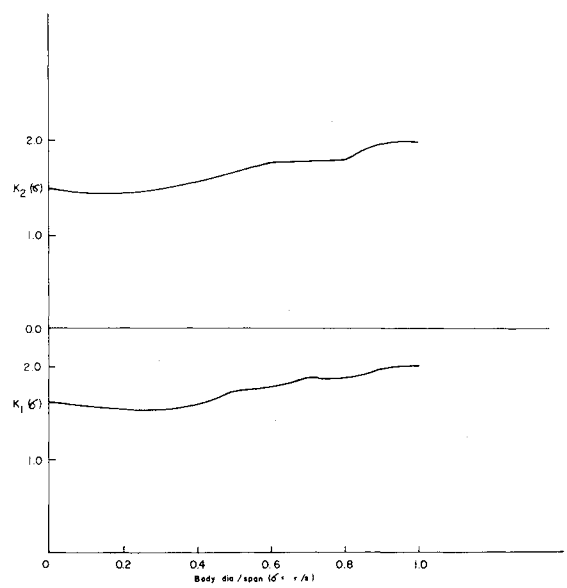

Fig. 1b Variation of factors K_l and K_2.

where the symbols have the usual meaning (Fig. 1a) and A' is the area of half of the fin-planform. For the supersonic case, ΔC_p is known from the linearized theory for planar wings and compiled[6] for arbitrary planforms. The integration in Eq. (1) can, therefore, be performed numerically for proper conditions of $\alpha(\xi)$ to obtain $C_{l\delta-}$ or C_{lp-}. For the subsonic case, an elliptic load distribution along the span is considered. Thus, from Eq. (1) one obtains

$$C_{l\delta-} = \frac{2}{SD}\int_r^s S(\xi)\left(\frac{\mathrm{d}CN_\alpha}{\mathrm{d}\xi}\right)_\xi \xi \mathrm{d}\xi$$

and

$$C_{lp-} = -\frac{4}{SD^2}\int_r^s S(\xi)\left(\frac{\mathrm{d}CN_\alpha}{\mathrm{d}\xi}\right)_\xi \xi^2 \mathrm{d}\xi \qquad (2)$$

where the products $S(\xi)(\mathrm{d}CN_\alpha/\mathrm{d}\xi)_\xi$ represent the elliptic spanwise loading over the fin such that its spanwise integration yields $CN_{\alpha f}$ based on the fin area ($2A'$). $CN_{\alpha f}$ is obtained[6] as

$$CN_{\alpha f} = 2\pi A \Big/ \left\{2 + \left[A^2\beta^2\left(1+\frac{\tan^2\Lambda}{\beta^2}\right)+4\right]^{1/2}\right\}$$

where A is the aspect ratio, Λ is the sweep-back angle, $\beta = (1-M^2)^{1/2}$, and M is the subsonic freestream Mach number.

Fin-Body Interference and Cruciform Effects

Interference Factor for C_{lp} and $C_{l\delta}$

Pitts et al.[7] have derived various formulae for calculating K_{wB} ($=k_{wB}+k_{Bw}$) based on the slender body and upwash theories. In the computation of C_{lp}, Barrowmann et al.[1] using upwash theory, have derived the following interference factor

$$KC_{lp} = \frac{\dfrac{\tau-\lambda}{\tau} - \dfrac{(1-\lambda)}{(\tau-1)}\log\tau}{\dfrac{(\tau+1)(\tau-\lambda)}{2} - \dfrac{(1-\lambda)(\tau^3-1)}{3(\tau-1)}} + 1 \qquad (3)$$

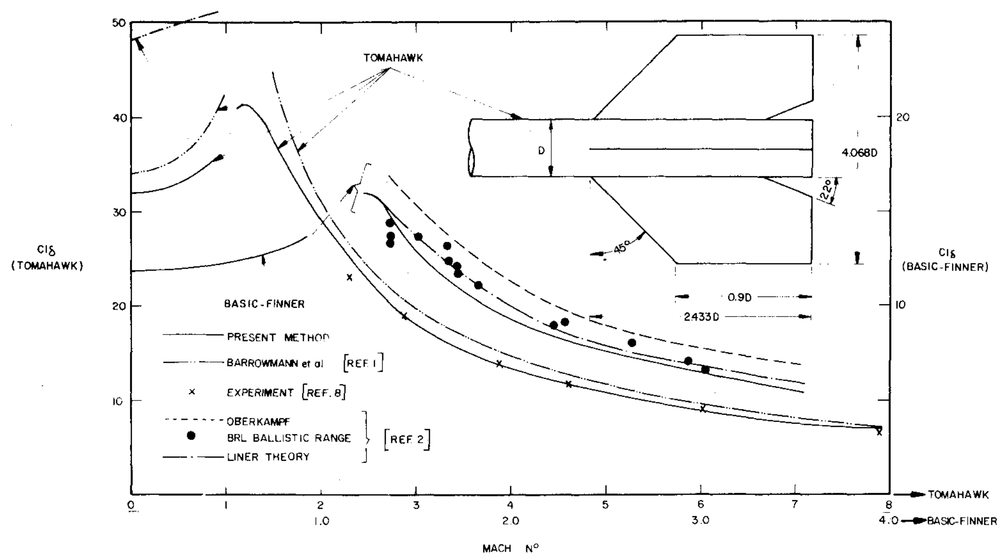

Fig. 2 Comparison of $C_{l\delta}$.

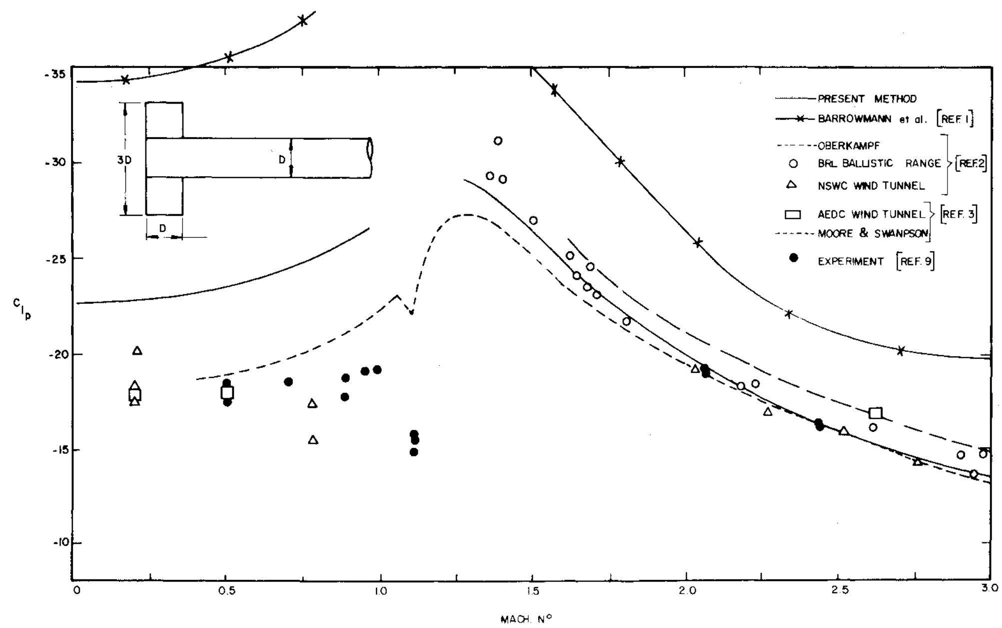

Fig. 3 Basic finner roll damping derivate.

where $\tau = s/r$ and λ is the taper ratio. In the present method, KC_{lp} is used for $\beta A > 2$ and K_{wB} (Ref. 7) for $\beta A \leq 2$, as it has been found[6] that upwash theory gives a better CN_α prediction only for $\beta A > 2$.

A judicious choice for the interference factor in the calculation of $C_{l\delta}$ is necessary. Oberkampf[2] has not considered such effects explicitly in his analysis at small incidence. Barrowmann et al.[1] have considered only k_{wB} in their calculation procedure. One should also consider k_{Bw} since the fin at incidence does influence the body to affect the moment. It can also be seen that for a similar situation in the calculation of C_{lp}, Eq. (3) is obtained from upwash theory and is equivalent to K_{wB}. Rollstin[4] has used K_{wB} as the interference in his analysis. In the present method, K_{wB} is used as the interference factor and, depending upon whether $\beta A \leq 2$ or $\beta A > 2$, K_{wB} is computed[7] from the slender body or upwash theory, respectively.

Effect of Cruciform Configuration

Adams and Dugan,[5] using slender body theory, investigated such effects on C_{lp} and $C_{l\delta}$ and concluded that

$$C_{lp+} = K_I(\sigma) C_{lp-} \quad \text{and} \quad C_{l\delta+} = K_2(\sigma) C_{l\delta-} \tag{4}$$

The factors K_I and K_2 depend upon the body diameter-to-span ratio, σ, and are presented in Fig. 1b, where this dependence is rather weak (up to a maximum of 6%) for $0 \leq \sigma \leq 0.3$ or 0.4. Generally, the value of σ is less than 0.4 for most of the sounding rockets, and K_I $(\sigma = 0) = 1.62$ and K_2 $(\sigma = 0) = 1.524$ can be used for a fin-body cruciform configuration. However, in the case of configurations with larger σ (small span) or otherwise, appropriate values of K_I and K_2 from Fig. 1b can be used. Now the total $C_{l\delta+}$ and C_{lp+} may be computed with the help of Eqs. (1-4).

Results and Conclusions

The present calculation method has been compared and validated[6] against a large set of experimental data, but due to space limitations, only a few sample comparisons are presented here in Figs. 2 and 3. In Fig. 2, $C_{l\delta}$ comparisons are shown for both Basic Finner and Tomahawk.[1,8] The present method can be seen to agree well with experiment in both the cases. The approach of Barrowmann et al.[1] appears to overpredict $C_{l\delta}$ highly for the Basic Finner in the subsonic region; that of Oberkampf also slightly overpredicts in the supersonic region.

C_{lp} results from the present method show very good agreement with experimental and flight data[2,3,9] (Fig. 3) in the supersonic regime. However, the present method does overpredict C_{lp} in the subsonic region. As can be seen in Fig. 3, the approach of Barrowmann et al.,[1] of course, seems to give a still higher prediction. It appears that the C_{lp} calculation is sensitive to the spanwise load distribution in the subsonic range, and that the present assumption of an elliptic distribution could be poor. However, it may be worthwhile analyzing experimentally whether the possibility of a lift loss can exist in the subsonic region in C_{lp} tests due to rotation of the model and thereby reducing C_{lp} appreciably.

Thus, it is brought out that the adaption of proper fin-body and fin-fin interference effects leads to a consistently better prediction of roll-rate derivatives. The subsonic region probably requires further investigation of the roll-damping derivative.

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